

Factors affecting liquefaction resistance and assessment by pore pressure model

Mitsu Okamura¹[0000-0002-9295-376X]

¹ Ehime University, Matsuyama, Ehime 790-8522, JAPAN
okamura@cee.ehime-u.ac.jp

Abstract. Assessment of the liquefaction resistance of clean sand still involves considerable uncertainties, which are a current research topic in the field of soil liquefaction. Factors to be considered includes shaking history overconsolidation, degree of saturation and partial drainage. The effects of these factors on liquefaction resistance have been studied in the laboratory and empirical relationships are derived. This paper describes the development of pore pressure generation model similar to that of Martin et al. [18] but based on stress-controlled triaxial tests. The effects of various factors on the pore pressure generation and liquefaction resistance of clean sand is explained using the unique index of volumetric strain. The model is verified through comparisons with the results of laboratory tests. It is confirmed that the plastic volumetric strain accumulated in sand either by drained or undrained loading dominates the increase in liquefaction resistance of pre-sheared, overconsolidated and unsaturated sand. The model provides a better understanding of the physical processes leading to liquefaction of saturated and unsaturated sand with and without stress histories.

Keywords: Excess Pore Pressure, Liquefaction Resistance, Sand, Volumetric Strain, Degree of Saturation, Stress History.

1 Introduction

Resistance of sand to liquefaction during undrained cyclic shearing has been investigated for several decades. Significant progress has been made in understanding the fundamental mechanism and influence of various factors on liquefaction resistance, including soil density, grain fabric, testing apparatus, stress anisotropy, all listed by Seed [23] (cited in Finn [5]). Although considerable research efforts have been devoted to the topic, assessment of liquefaction resistance of clean sand still involves considerable uncertainties and the effects of influential factors are a current topic in the field of soil liquefaction. Factors include loading history (shaking history and over consolidation), degree of saturation and partial drainage. The effects of these factors on liquefaction resistance have been examined in several studies, mostly in the laboratory.

With regard to the effect of shaking history, it is recognized that sand in seismically active areas has a higher resistance to liquefaction than that in calm areas. Small pre-shearing events during earthquakes are believed to make the sand more resistant to liquefaction (Dobry and Abdoun [2], and Towhata et al. [31]). Observations based on field

evidences have been corroborated by laboratory test results. It has been reported that cyclic pre-shearing enhanced the liquefaction resistance of sand, first by Finn et al. [4], followed by numerous researchers including Singh et al. [26], Tokimatsu and Hosaka [29], and more recently Kiyota et al. [15], Goto and Towhata [7], Wu et al. [36], Toyota and Takada [32], Nelson and Okamura [19] and Wu and Kiyota [37]. For example, Tokimatsu et al. [30] conducted cyclic triaxial tests on medium dense and dense Niigata sand and found that drained pre-shearing of up to ten thousand cycles tripled the liquefaction resistance, even though the increase in the relative density was only up to several per cent. Results of similar tests in the literature consistently indicate that the liquefaction resistance increases with the number of cycles and cyclic stress ratio of the pre-shearing. However, none of the testing parameters alone quantified the improvement of the liquefaction resistance. Finn et al. [4] reported that pre-shearing with shear strain amplitude larger than 0.5% decreased rather than increased liquefaction resistance. This contradictory results of the effect of pre-shearing add further complexity. Further studies sought to define the separation between beneficial and deleterious pre-shearing (Ishihara and Okada [12], Suzuki and Toki [27], Nelson and Okamura [19]). The loading history (i.e., over-consolidation ratio) has a substantial effects on liquefaction resistance. The effect of the overconsolidation ratio (OCR) and the value of the lateral pressure coefficient at rest (K_0) were separately investigated. The OCR effect was demonstrated experimentally by Seed and Peacock [24] and confirmed later by Ishibashi and Sherif [9]. Experimental data has been further accumulated (Ishihara and Takasu [13], Kokusho et al. [16], Tatsuoka et al. [28] and Toyota and Takada [32]) and an empirical equation of liquefaction resistance was proposed as a power function of OCR (Kokusho [17]).

The liquefaction resistance of unsaturated sand has been studied in the laboratory. In early research, the degree of saturation of the tested specimens was generally close to 100%, because the primary objective in the studies was to establish a standard for laboratory cyclic shear tests, and it was therefore necessary to avoid undesirable unsaturated conditions that would result in overestimation of the liquefaction resistance. Thereafter, unsaturated sand with a lower degree of saturation, down to approximately 70%, has been tested by several researchers. The results have been expressed in the relationship between the degree of saturation and the liquefaction resistance of unsaturated sand normalized with respect to that of fully saturated sand (Huang et al. [8], Yoshimi et al. [34], Yasuda et al. [33], Ishihara et al. [14] and Goto and Shamoto [6]). Extensive data accumulated so far conclusively indicates that the liquefaction resistance ratio increases with decreasing degree of saturation. However, the liquefaction resistance ratios were considerably different for different sands tested under different conditions, indicating that the degree of saturation is not the only parameter affecting the normalized liquefaction resistance ratio of unsaturated sand.

Parameters such as the number of cycles and cyclic stress ratio of pre-shearing, over consolidation ratio (OCR), and degree of saturation are all influential; however, none of these are sufficient to explain the nature of sand behavior. In recent years, attempts have been made to explain the effects of those factors through a unified single parameter of volumetric strain. For unsaturated sand, volumetric strain calculated according to the increase in excess pore pressure has been found to be a dominant factor in

liquefaction resistance (e.g., Okamura and Soga [20] and Okamura and Noguchi [21]). The liquefaction resistance ratio uniquely correlates with the volumetric strain during undrained cyclic shearing. Regarding pre-shearing effects, volumetric strain accumulated by small shakings is also a factor that can explain the enhancement of liquefaction resistance during a subsequent undrained shaking event (Okamura et al. [22] and Nelson and Okamura [19]).

The principal mechanics of pore pressure buildup and liquefaction have emerged from a finding made early in the history of liquefaction study by Martin et al. [18]. They related the volumetric densification of sand, when subjected to cyclic shear loading in drained condition, to the increase in pore pressure that occurs when the same cyclic loading is applied to the saturated sand in undrained condition. The plastic volumetric strain due to the densification of the sand skeleton is cancelled by the elastic volumetric strain to maintain a constant sand volume, and the elastic strain can be correlated with the increase in pore pressure. The validity of this model showing the relationship between densification and pore pressure buildup was verified by several laboratory studies in the 1970s and 1980s. Martin et al. [18] and Finn [5] proposed a pore pressure generation model based on volumetric strain evolution under constant cyclic strain tests using simple shear apparatus. Their model successfully explained the effects of stress and strain history as well as the degree of saturation on the excess pore pressure generation in undrained cyclic shearing. Although the basic concept is simple and clear, its practical application is difficult because many experimental parameters are involved in the model, some of which are determined by strain-controlled simple shear tests instead of the stress-controlled tests commonly used in practice. Changes in horizontal earth pressure in the simple shear apparatus add further complexity to the model. Nevertheless, it has been established that volumetric plastic strain characteristics in conjunction with elastic stiffness are the properties determining the excess pore pressure not only for freshly deposited and reconstituted sand but also for sand with stress and strain history and for unsaturated sand.

This paper describes a pore pressure generation model based on cyclic stress ratio that is similar to that of Martin et al. [18] but based on a stress-controlled triaxial test. Attempts are made to explain the effect of various factors on the pore pressure generation and liquefaction resistance of clean sand using the index of volumetric strain. The model is verified through a comparison with the results of laboratory tests. Note that effect of partial drainage is being investigated in ongoing research and has not been included in this paper but will be presented elsewhere in the near future.

2 Stress-Based Pore Pressure model

2.1 Pore pressure model

The basic mechanics of densification and pore pressure generation in the undrained cyclic shearing were conceptually demonstrated by Martin et al. [18]. Volumetric strain (densification) of sand consists of recoverable strain stored by elastic deformation at grain contacts (ε_{ve}) and irrecoverable strain due to slippage at the contacts (ε_{vp}). ε_{vp} is

directly associated with ε_{ve} ($= -\varepsilon_{vp}$) and is converted into a change in effective stress if a constant volume condition is imposed. This is explained in a conceptual and rheology model in Fig. 1. Slippage at soil grain contacts occurs when subjected to cyclic shearing, which is the cause of plastic volumetric strain, ε_{vp} , — indicated as the distance between points A and B in Fig. 1(a) and measured by a plastic slider in Fig. 1(b). When the undrained condition is imposed, ε_{vp} has to be compensated by the same amount of negative volumetric strain,

$$\varepsilon_{ve} + \varepsilon_{vp} = 0 \quad (1)$$

In other words, the soil grain structure rebounds to the extent required to keep the volume constant—indicated from points B to C on the rebound line. With the bulk modulus of water, K_w , being sufficiently high to ignore the volume change of water, change in pore pressure can be expressed using the elastic (rebound) modulus of sand, E_r , as

$$\Delta u = -\Delta\sigma' = \varepsilon_{vp} \times E_r. \quad (2)$$

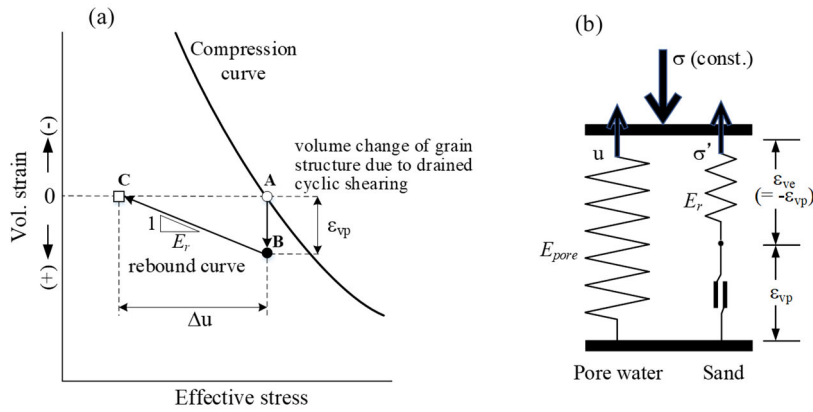


Fig. 1. Densification in undrained cyclic shearing and generation of pore pressure.

Silver and Seed [25] performed cyclic simple shear tests and found that there is a unique relationship between ε_{vp} and cyclic shear strain amplitude for a particular density of sand at a given number of cycles irrespective of the effective confining stress, indicating that volumetric strain is uniquely related to cyclic shear strain. Martin et al. [18] and Finn [5] modeled the evolution of ε_{vp} as a function of number of cycles and shear strain amplitude based on drained cyclic simple shear tests where specimens were subjected to cycles of constant shear strain amplitude.

To estimate volume changes during the stress-controlled cyclic shear tests commonly conducted in engineering practice and for the field liquefaction assessment for a specified earthquake acceleration time history, the shear stress has to be converted to corresponding shear strain in order to apply the strain-based model. This can be done using shear modulus; however, shear modulus is expressed by non-linear functions of shear strain and effective confining pressure. The model of Martin et al. [18] and Finn [5] has many parameters to be determined experimentally. It is clear the volumetric change is

more closely related to cyclic shear strain than to cyclic shear stress and that the strain-based ϵ_{vp} model is appealing. However, the advantage of the model that it does not use the effective confining stress is negated when introducing shear modulus to convert shear stress into shear strain. Moreover, the confining pressure effects that were found to be negligible by Silver and Seed [25] and Youd [35] have a substantial influence on the volumetric strain behavior of sand tested in a simple shear apparatus (Duke et al. [3]). Nevertheless, the model provided a clear basis for better understanding the physical processes of progressive pore water pressure increase during undrained cyclic shearing, leading to liquefaction.

2.2 Densification due to constant stress cycles

Although shear strain is more clearly correlated with plastic volumetric strain than is shear stress, a stress-based model has an apparent advantage over the shear strain-based model, namely that in liquefaction triggering assessment, acceleration and thus shear stresses are directly used to obtain volumetric strain in the stress-based model. In addition, simple shear testing used by Martin et al. suffers possible influences of horizontal stress change. In order to avoid further complication associated with change in horizontal stress, triaxial testing is employed to build a stress-based model in the present study.

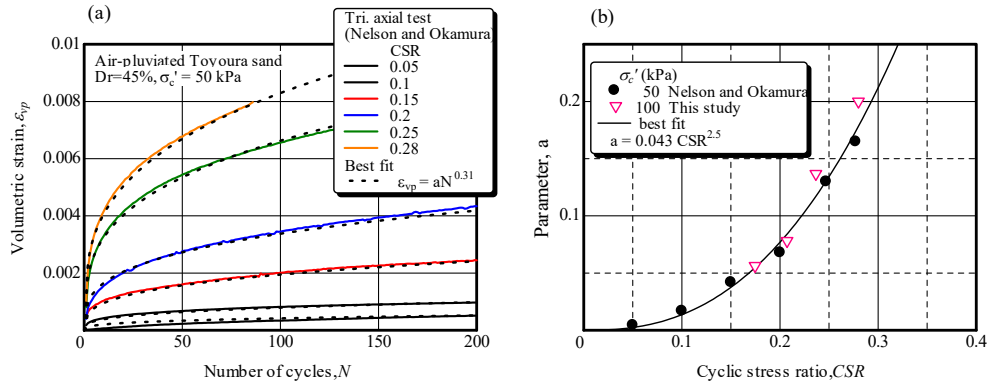


Fig. 2. Plastic volumetric strain. (a) Evolution of volumetric strain of loose Toyoura sand in drained cyclic shearing together with best fitted curves. (b) Variation of parameter a with CSR.

The sand used for all the tests conducted and described in this study to develop and verify the pore pressure model was medium dense Toyoura sand. The specific gravity and minimum and maximum void ratio of the sand are $G_s = 2.64$, $e_{min} = 0.609$, and $e_{max} = 0.973$, respectively. Specimens were air-pluviated medium dense Toyoura sand in a range of relative density $Dr = 45 - 51\%$, confined isotopically at effective stresses of 50 kPa and 100 kPa. Since all the specimens were prepared using the same sand and method, the effects of sand type, relative densities and sand fabric are not discussed in this paper.

Fig. 2(a) indicates the evolution of volumetric strain with the number of uniform shear stress cycles in drained triaxial tests, where the volumetric strain at the end of each cycle is plotted. The rate of increase in ε_{vp} is higher for a higher cyclic stress ratio (CSR), however, the curves are geometrically similar. All the curves are well approximated with a function of the form

$$\varepsilon_{vp} = a \times N^b \quad (3)$$

where a and b are parameters and $b = 0.31$ gives a good fit to the experimental data over the range up to 200 cycles. The parameter a is plotted in Figure 3 and can be well expressed by the following function, irrespective of the confining pressures:

$$a = c \times \text{CSR}^d \quad (4)$$

A set of three parameters can be used to completely defines the volume change behavior under drained cyclic shearing of constant CSR: $(b, c, d) = (0.31, 0.039, 2.5)$. It should be noted that there is a limiting shear strain below which no pore water pressure develops regardless of the number of loading cycles. This value is of the order of $10^{-2}\%$ (Dobry et al. [1]). Conversely, as shown by equations (2) and (3), plastic volumetric strain occurs for any cyclic stress ratio. However, strain in the range corresponding to a very small CSR, less than approximately 0.05 for the particular soil, may not generate significant excess pore pressure leading to soil liquefaction, and is not considered in this study.

Duke et al. [3] reported that cyclic volumetric strain decreased with increasing confining pressure, based on simple shear tests at a constant shear strain amplitude. This is more significant for a higher confining pressure range (>100 kPa). Possible reasons for the results in Fig. 2(b), which does not show a dependency on confining stress, are that the stress range tested in this study was relatively low (< 100 kPa) and narrow (50 – 100 kPa), and a different test apparatus was used. Further research is apparently needed to investigate the stress level dependency of volumetric strain on stress level.

2.3 ε_{vp} due to random stress cycles

Cycles of constant shear stress amplitude, τ_d , are applied to specimens in most common laboratory liquefaction tests in which the cyclic shear stress ratio defined as $\text{CSR} = \tau_d / \sigma_{c0}'$ is constant throughout the test, and σ_{c0}' denotes the initial effective confining pressure. However, the actual cyclic stress ratio, $\text{CSR}_a = \tau_d / \sigma_c'$, where σ_c' is effective stress at a given time, is not constant but increases due to the generation of excess pore pressure, and thus decreases the effective stress. This means that in order to estimate plastic volumetric strain for sand in undrained cyclic shearing from the ε_{vp} model (equations (3) and (4)), the value of CSR_a in each cycle should be considered even for a constant CSR test. Variation in actual cyclic stress ratio from cycle to cycle is also the case when random shear stress caused by earthquake acceleration is considered.

It is clear that sand that has already been subjected to cycles of shearing and therefore has accumulated ε_{vp} shows a less contractive response to cyclic shearing than that of freshly deposited sand without any accumulated ε_{vp} . A basic assumption similar to that

used by Martin et al. [18] is employed in this study: plastic volumetric strain increment for a cycle of shearing, $\Delta\varepsilon_{vp}$, depends on CSR_a and plastic volumetric strain accumulated since the deposition of the sand, ε_{vp} . Independent of the number of cycles and the CSR previously experienced, ε_{vp} represents the densification characteristics of sand. For sand accumulating ε_{vp} , the plastic volumetric strain increment, $\Delta\varepsilon_{vp}$, for a cycle of CSR_a can be obtained by finding the equivalent number, N_e , for the ε_{vp} on the CSR_a curve, and then reading $\varepsilon_{vp} + \Delta\varepsilon_{vp}$ corresponding to $N = N_e + 1$, as shown in Fig. 3. Freshly deposited sand without ε_{vp} has the highest potential to densify and $\Delta\varepsilon_{vp}$ decreases as the accumulated plastic strain increases.

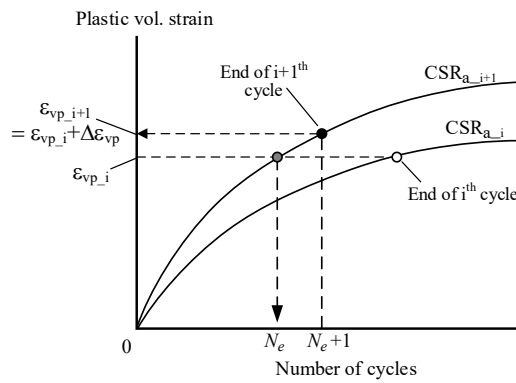


Fig. 3. Dependency of volumetric strain increment ($\Delta\varepsilon_{vp}$) on accumulated plastic strain (ε_{vp}).

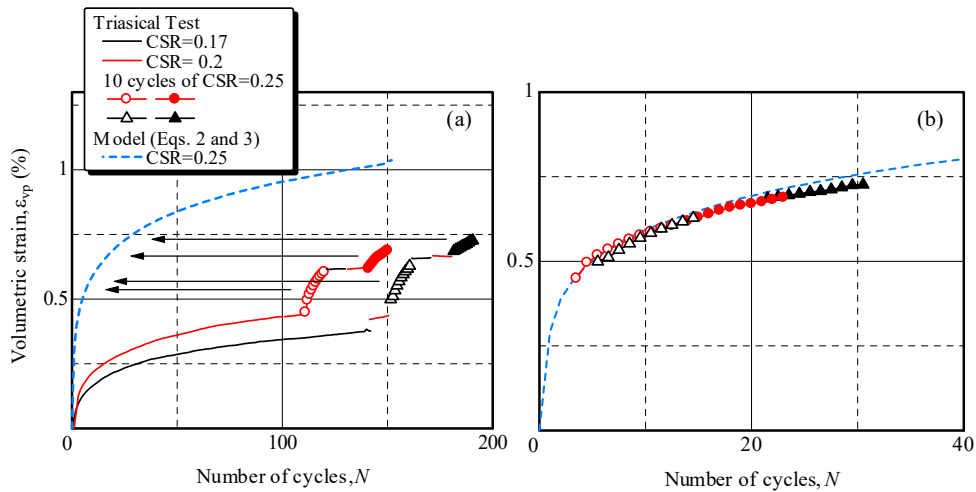


Fig. 4. Volumetric strain behavior for tests with constant CSR and varying CSR.

Figure 4 shows the results of tests conducted to verify this assumption. Medium dense Toyoura sand specimens were subjected to an initial confining pressure of 100

kPa. Two specimens were subjected drained cyclic shearing with a CSR of 0.17 and 0.2 until an ε_{vp} value of approximately 0.4% was attained, followed by an additional 10 cycles of CSR= 0.25, 0.2 r 0.17. As indicated in Figure 4(b), sections of additional 10 cycles of CSR=0.25 (indicated by circles and triangles in the figure), when horizontally shifted leftward falls on the model curve of the corresponding CSR_a, showing validity of the assumption. The effect of ε_{vp} accumulated in sand after deposition, based on, volumetric densification characteristics, is hereafter referred to as the ε_v effect.

2.4 Elastic bulk modulus

The elastic (rebound) modulus of sand, E_r , was observed in triaxial tests. Specimens were isotopically consolidated at $\sigma'_v=100$ kPa, followed by unloading to 30 kPa and reloading back to 100 kPa, before and after drained cyclic shearing. A typical response is shown in Fig. 5. Drained cyclic shearing, which caused 0.6% plastic volumetric strain, did not alter the elastic modulus. Although the rebound curves were non-linear, a linear approximation was adopted for simplicity in this study. The curve was reasonably approximated by with lines of $E_r = 24.0$ MPa and 16.0 MPa for the ranges from 100 kPa to 50 kPa and 50 kPa to 25 kPa, respectively, which have been used in subsequent sections to estimate excess pore pressure. It is reasonable to use these E_r values for ranges of higher effective stress, from 100 to 50 kPa for specimens with $\sigma_{c0}'= 100$ kPa and from 50 to 25 kPa for those with $\sigma_{c0}'= 50$ kPa, in order to estimate pore pressure behavior in liquefaction tests. This is because, as cyclic shearing proceeds and the effective stress decreases, CSR_a will be dominant in the estimation of Δu increment. E_r is more important in the early stages of the liquefaction test.

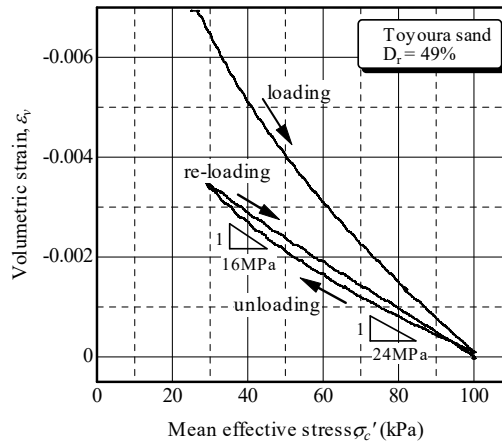


Fig. 5. Elastic rebound during unloading.

3 Excess Pore Pressure Buildup in Undrained Cyclic Shearing

Triaxial liquefaction tests were conducted on medium dense Toyoura sand. All specimens were fully saturated with the Skempton's B value higher than 0.95 and confined at $\sigma_{c0}' = 100$ kPa. Fig. 6 shows typical results of the observed excess pore pressures ratio (EPPR) during cyclic shearing. This was also simulated using the pore pressure model as shown in the figure. The excess pore pressure ratio increment for a single cycle in the triaxial test is large for the first cycle, decreased gradually with the number of cycles and increased sharply after the excess pore pressure ratio (EPPR) reached approximately 0.6. Finally, the EPPR jumped up to 100% shortly after the effective stress path reached the phase transformation line (Ishihara et al. [10]). It can be seen that the overall trend of EPPR evolution for the test observations and the pore pressure model estimations agree quite well. The rapid increase in estimated excess pore pressure after the EPPR exceeded 0.6 is also evident, although the model did not account for the abrupt change in sand behavior after the phase transformation line was reached. This is attributed to the sharp increase in CSR_a . The plastic strain increment ($\Delta\varepsilon_{vp}$) and actual cyclic stress ratio (CSR_a) in each cycle estimated from the model are shown in Fig. 7. The plastic strain increment in one cycle ($\Delta\varepsilon_{vp}$) increased with CSR_a particularly after EPPR exceeded approximately 0.6.

The liquefaction strength curve for the sand is shown in Fig. 8. Open circles show the CSR versus the number of cycles to reach the liquefaction condition for four liquefaction tests. The solid line corresponds to the liquefaction resistance curve estimated with the pore pressure model, which predicts the test results reasonably well.

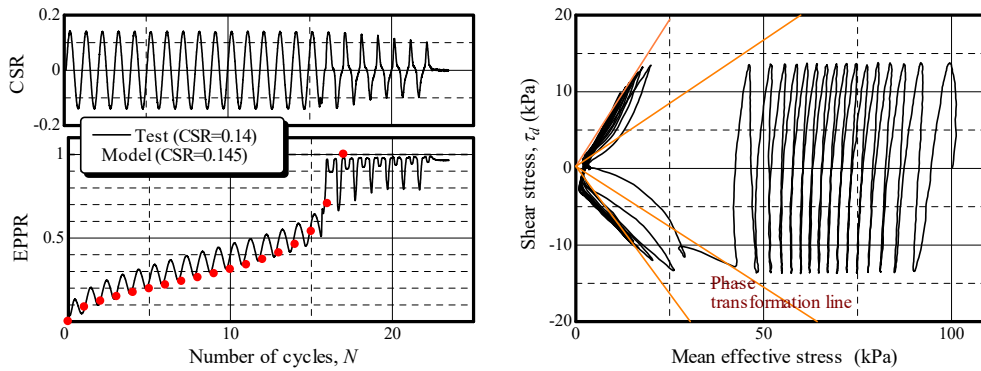


Fig. 6. Liquefaction test results on medium dense Toyoura sand ($D_r=46\%$).

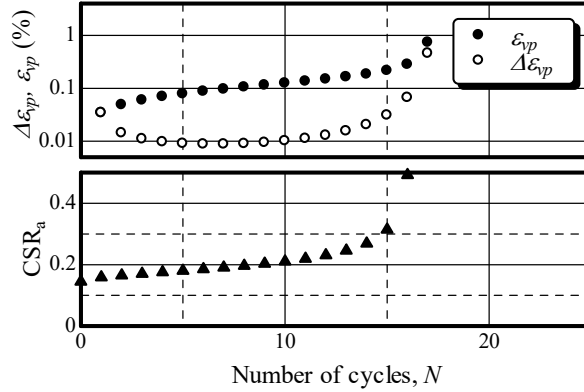


Fig. 7. Variation in plastic volumetric strain and actual cyclic stress ratio.

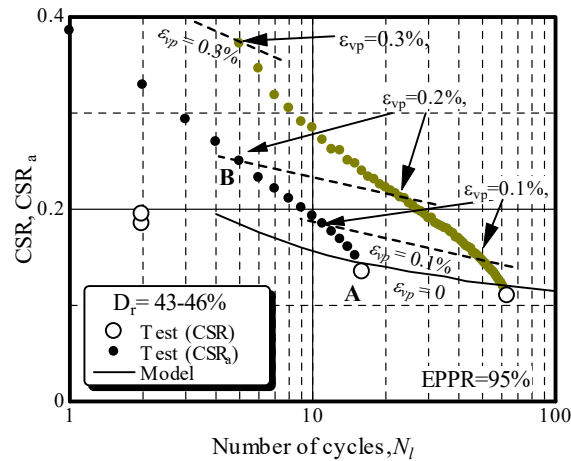


Fig. 8. Relationship between CSR and number of cycles to liquefy.

In the tests, the actual cyclic stress ratio increased with an increase in excess pore pressure and with the number of cycles. The filled circles are the plots of CSR_a and the remaining number of cycles to liquefaction from that point. For instance, point A in the figure corresponds to a specimen before shearing, which liquefied in $N_l = 16$ cycles of uniform CSR of 0.135. Filled circles indicate the relationship between CSR_a in each cycle and the remaining number of cycles needed to liquefy the specimen, $N_l - N$. At $N = 10$, the excess pore pressure was approximately 42 kPa and CSR_a was 0.23 ($=0.135/(100-42)$). This indicates that the specimen at this time withstood 6 cycles of $CSR = 0.23$, which shows higher resistance to liquefaction than that of freshly deposited medium dense Toyoura sand specimen consolidated at 58 kPa. The difference between CSR_a and CSR for a given number of cycles in this figure represents an increment in resistance to liquefaction which was gained due to cyclic shearing. During cyclic

shearing, even in an undrained condition, sand gradually accumulates ε_{vp} and decreases its potential for volumetric strain. The estimated ε_{vp} using observed excess pore pressure and equation (1) are shown in Fig. 8. Equi-strain conditions are depicted with broken lines.

4 Influence of Factors on Excess Pore Pressure Generation and Liquefaction resistance

4.1 Small Pre-shearing

In order to study the effects of pre-shearing, a series of triaxial tests was conducted by Nelson and Okamura [19]. Stress controlled cyclic shearing was applied to specimens in drained condition first, and undrained cyclic shearing was then applied to observe the excess pore pressure generation and resistance to liquefaction. For the pre-shearing, the cyclic stress ratio and number of cycles were systematically varied in the tests to achieve the target volumetric strains of $\varepsilon_{vp_ps} = 0.1\%$, 0.3% or 0.8% . The cyclic stress ratios applied in the pre-shearing were selected in order to limit the shear strain double amplitude in all cycles to be lower than 0.35% , which is believed to be lower than the threshold shear strain beyond which pre-shearing becomes deleterious. More detailed test conditions and results can be found in Nelson and Okamura [19]. Typical responses to the undrained cyclic shearing of specimens with different target volumetric strains ($\varepsilon_{vp_ps} = 0, 0.1\%$, and 0.3%) are shown in Fig. 9. For the same CSR, specimens accumulating higher ε_{vp_ps} required more cycles to liquefy compared to that without pre-shearing. The cyclic stress ratio and number of cycles, N_l , to reach an excess pore pressure ratio (EPPR) of 95% for all the tests are shown in Fig. 10. The pre-shearing enhances the liquefaction resistance significantly, although the increase in the relative density due to pre-shearing is marginal. A volumetric strain of 0.8% corresponds to only a 3.9% increase in the relative density. The data points corresponding to the same ε_{vp_ps} lie almost on the same line irrespective of the CSR and number of cycles in the antecedent pre-shearing. The cyclic stress ratio to cause liquefaction for a given number of cycles increases with increasing ε_{vp_ps} , confirming that the volumetric strain in pre-shearing dominates the pre-shearing effects on the liquefaction resistance, and that the effects of the CSR and the number of cycles are negligible.

The pore pressure model was used to predict the pore pressure response for the pre-sheared sand. The primary effect of pre-shearing is to reduce the volume strain potential. The accumulated volumetric strain during the pre-shearing, ε_{vp_ps} , was reflected in the model calculation by introducing ε_{vp_ps} as the initial volumetric strain condition. Typical excess pore pressure responses are depicted in Fig. 9 for the three specimens, which accumulated ε_{vp_ps} of either $0, 0.1\%$ or 0.3% and were subjected to undrained cyclic shearing with a similar CSR. The model simulated the observed excess pore pressure generation quite well. It is of interest that excess pore pressure in the tests increased more or less linearly with the number of cycles for pre-sheared sand, while for the virgin sand, the pressure increase in the first cycles was large and slowed down

in the following cycles. This pattern of excess pore pressure response is reasonably well represented by the model.

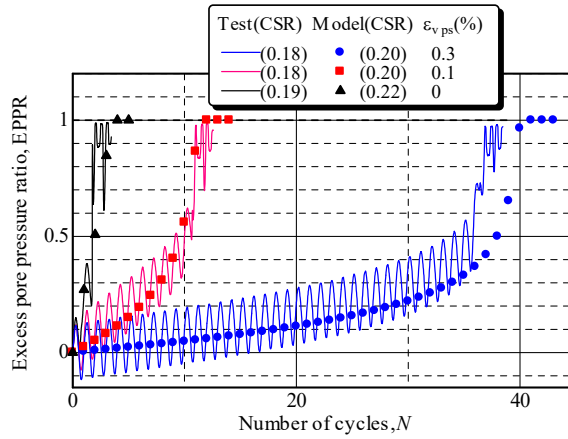


Fig. 9. Excess pore pressure ratio for different volumetric strain by pre-shearing

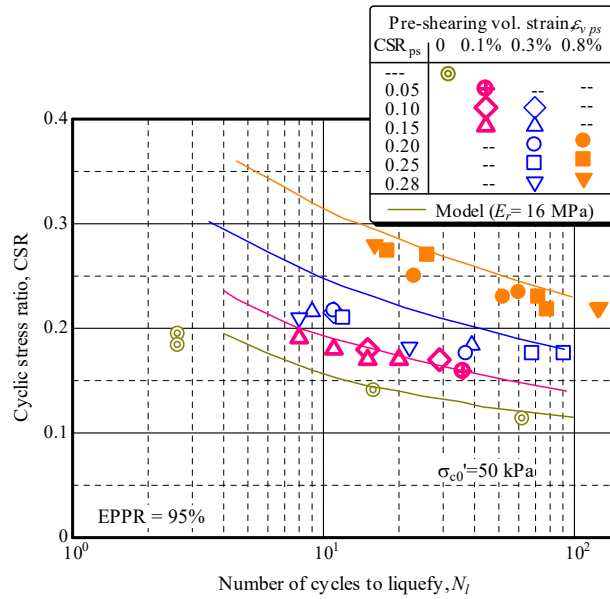


Fig. 10. Liquefaction resistance for different volumetric strain by pre-shearing

The solid lines in Fig. 10 show the liquefaction resistance curves for pre-sheared sand with different $\epsilon_{v,ps}$ estimated with the pore pressure model. The model successfully predicted the liquefaction curves for pre-sheared medium dense Toyoura sand. It

should be noted that the lines for $\varepsilon_{vp_ps} = 0.1\%$ and 0.3% in this figure almost coincide with those for $\varepsilon_v = 0.1\%$ and 0.3% in Fig. 8. Sand subjected to undrained cyclic shearing accumulates plastic volumetric strain in accordance with generated excess pore pressure as shown in Fig. 1 and equation (2) and the pre-shearing volumetric strain has the same effect in reducing the potential for further densification.

4.2 Degree of Saturation

Unsaturated sand can change its volume even if undrained condition is imposed. The volume change of an unsaturated sand mass in undrained condition is equal to that of its void, which is comprised of not only incompressible water but a water-air mixture. The volumetric constraint shown by equation (1) for fully saturated sand in undrained condition can be extended for unsaturated sand as,

$$\varepsilon_v = \varepsilon_{vp} + \varepsilon_{ve} \quad (5)$$

where ε_v denotes the volumetric strain of the sand mass, which is purely due to the compression of air existing as a part of the pore fluid in the sand. It is reasonable to assume that the pore air pressure is the same as the pore water pressure for sand at relatively high degree of saturation, 90% or higher, where the air bubbles exist separately in a form of insular saturation. The difference between the air bubble pressure and the surrounding water pressure given by the Young-Laplace equation is, for a spherical bubble,

$$p_{air} - p_{water} = 2T_s / r \quad (6)$$

where p_{air} is the pressure inside the air bubble, p_{water} is pressure in the surrounding water, T_s is the surface tension, and r is the bubble radius. Using a surface tension for the air-water interface of 0.075 N/m , the excess pressure in a bubble of radius 0.5 mm is 0.6 kPa , which is small compared to the pore pressure discussed and can be considered negligible. The value of ε_v that arises from excess pore pressure Δu is given as

$$\varepsilon_v = \Delta u \cdot n / K_{pf} \quad (7)$$

where K_{pf} and n are the bulk modulus of pore fluid and the porosity, respectively. From equations (2), (5) and (7),

$$\Delta u = \frac{K_{pf} E_r}{K_{pf} + n E_r} \Delta \varepsilon_{ve} \quad (8)$$

Ignoring the volume change of water and assuming the Δu in each cycle of shearing is relatively small, K_{void} can be expressed using Boyle's law for an ideal gas as

$$K_{pf} = p / (1 - S_r) \quad (8)$$

where S_r is the degree of saturation, and p is absolute pore pressure (Okamura and Soga [20]). Fig. 11(a) depicts the volumetric strain-effective stress plane, on which paths are indicated for fully saturated sand in undrained and drained shearing, as well as for unsaturated sand in undrained shearing. A cycle of given actual stress ratio CSR_a yields plastic strain ε_{vp} indicated from points A to B, which is converted to the pore pressure increase Δu (B to C) if the sand is fully saturated and in undrained condition as described in Fig. 1. For unsaturated sand, the generated pore pressure and resultant volumetric strain is defined as the intersection point D of two lines (equations (7) and (8)). Fig. 11(b) presents the results of undrained cyclic tests for three different degree of saturation, $S_r=100, 98$ and 95.5% , subjected to CSR in a small range of $0.18 - 0.21$ (Okamura and Soga [20]). Plastic strains derived from the model are also indicated by crosses. The numbers shown in this figure correspond to the number of cycles of shearing. In comparing the points at the end of the first cycles labeled “1”, the CSR_a of the tests are approximately the same, and the excess pore pressure decreases with decreasing S_r . Δu was reduced by more than 80% by decreasing S_r from 100% to 95.5%.

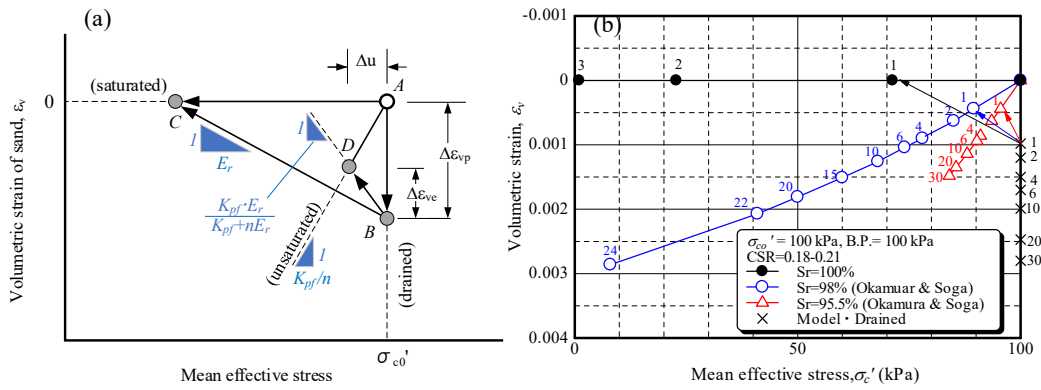


Fig. 11. Volumetric strain in undrained cyclic shearing and generation of pore pressure for saturated and unsaturated sand.

Figure 12 shows the effect of the degree of saturation on the relationship between cyclic stress ratio and the number of cycles required to liquefy medium dense Toyoura sand, from results obtained in triaxial tests (Okamura and Soga, 2006). Both the initial effective confining pressure and the back pressure were 100 kPa and the degree of saturation was set to either 100%, 98% or 96%. The tests were also simulated using the pore pressure model, with the results indicated by solid lines. The bulk modulus of the fluid was set at $K_{pf} = 11.0$ MPa and 5.1 MPa for $S_r = 98\%$ and 96%, respectively. The test results shows that lowering the degree of saturation by 4% almost doubled the liquefaction resistance and the model simulated the liquefaction resistance curves quite well.

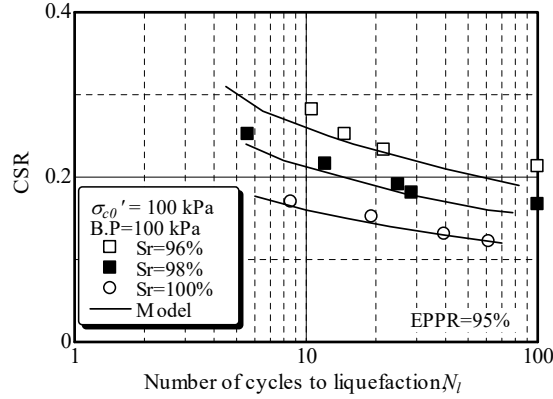


Fig. 12. Liquefaction resistance curves for saturated and unsaturated sand.

4.3 Overconsolidation

Undrained cyclic triaxial tests on normally consolidated and overconsolidated medium dense Toyoura sand were conducted, and the results are shown in Fig. 13. Figure 14 summarizes the liquefaction resistance of overconsolidated medium dense Toyoura sand as reported in the literature. The results of the present study, Tatsuoka et al. [28] and Toyota and Takada [32] are quite consistent; all the data is plotted in a narrow range which shows the liquefaction resistance increasing linearly with logarithm of OCR.

A loading-unloading cycle adds plastic volumetric strain to the sand. Figure 15 shows medium dense Toyoura sand at a confining pressure of 100 kPa with an overconsolidation ratio (OCR) of 2 and 4 accumulated ε_{vp} of 0.18% and 0.38%, respectively.

It should be noted that the rebound modulus E_r in the range from 100 to 50 kPa significantly increased with increasing OCR. Duke et al. [3] found that the overconsolidation increased the modulus but its effect was not significant for higher overburden stress (100 kPa). Effects of isotropic loading-unloading cycles on E_r may be somewhat different from that of the pre-shearing, which did not alter E_r . The appropriate values of E_r to be used in the following simulation are, for sand of OCR=2 and 4, 38 MPa and 49 MPa, respectively.

Since the value of ε_{vp} at the beginning of the undrained cyclic shearing was known, the relationship between CSR and the number of cycles required to liquefy the sand was estimated using the water pressure model, as shown in Fig. 13. In the model, overconsolidated sand is treated in the same way as pre-sheared sand which accumulated ε_{vp} prior to the undrained cyclic shearing. The results of simulation using the pore pressure model shown in Figs. 13 and 14 compare well with test results.

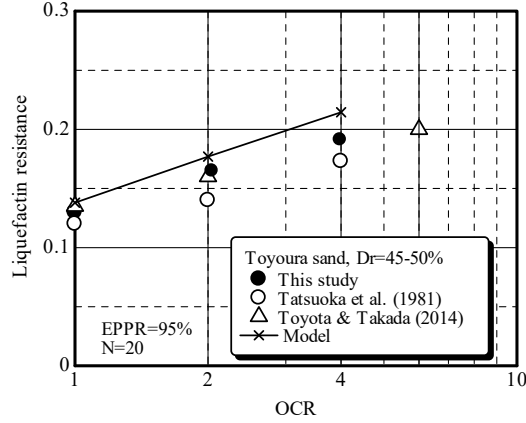


Fig. 13. Liquefaction resistance curves for overconsolidated Toyoura sand.

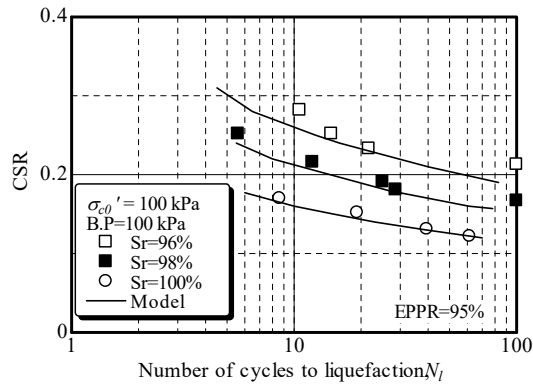


Fig. 14. Effect of OCR on liquefaction resistance (CSR to reach EPPR of 95% in 20 cycles).

5 Conclusions

This study developed a pore pressure model based on stress-controlled triaxial tests. The model follows a basic assumption in Martin et al. [18] that a unique relationship exists between volumetric strains in drained tests and excess pore pressure in undrained tests. The plastic volumetric strain developed during undrained cyclic shearing is absorbed by elastic rebound in the soil skeleton due to the reduction in effective stress, preserving the condition of constant volume. The plastic volumetric strain in drained tests was modeled based on constant CSR triaxial tests. This cyclic stress-based model has an apparent advantage over the shear strain-based model of Martin et al. in two aspects. In the assessment of liquefaction triggering conditions, acceleration and thus shear stress is directly used to obtain volumetric strain in the stress-based model, while strains must be converted into stresses using shear modulus in the strain-based model.

Simple shear testing used in the model of Martin et al. experiences possible influences from complicated variations in horizontal stress, whereas triaxial testing is free from this issue.

The effects of various factors on the pore pressure generation and liquefaction resistance of clean sand are explained using the unique index of volumetric strain through triaxial tests and simulation using the proposed model. The major findings are summarized as follows.

- Progressive pore pressure buildup during undrained cyclic shearing in saturated or unsaturated sand with and without stress histories was investigated with triaxial tests and the pore pressure model. There was a general trend of excess pore pressure buildup observed in triaxial tests in sand without pre-shearing and loading history, with the rate of pore pressure generation decelerating in the early stages and accelerating in the later stages with the number of cycles. This is attributed to the ε_{vp} accumulation and the increase in actual cyclic stress ratio (CSR_a). The accumulation of ε_{vp} , which decreases the potential for volumetric strain, has the beneficial effect of enhancing resistance to the generation of excess pore pressure (ε_{vp} effect), whereas the increase in actual cyclic stress ratio due to the generation of excess pore pressure has a detrimental effect (CSR_a effect).
- The excess pore pressure observed in triaxial tests is the result of the combined effects of accumulated ε_{vp} and CSR_a . Comparisons of triaxial specimens at the same excess pore pressure, and thus the same ε_{ve} , with specimens having the same ε_{vp} due to pre-shearing makes it possible to better understand the individual effects of ε_{vp} and CSR_a .
- A small pre-shearing has a beneficial effect in enhancing resistance to pore pressure generation and liquefaction resistance of sand. This can be explained by the ε_{vp} effect accumulated prior to the undrained cyclic shearing. Plastic volumetric strain accumulated in sand due to undrained cyclic shearing and that due to pre-shearing have the same effect in reducing the potential for further densification.
- Unsaturated sand changes its volume during cyclic shearing even if undrained condition is imposed. Volumetric compatibility and the bulk modulus of pore fluid (water-air mixture) are considered in the pore pressure model. The model simulates pore pressure generation and liquefaction resistance curves of triaxial tests reasonably well.
- Overconsolidated sand has a higher resistance to liquefaction than normally consolidated sand. This is explained by the plastic volumetric strain and increase in the rebound modulus caused in the loading-unloading process.

The proposed pore pressure model simulated the buildup of pore pressure during cyclic shearing. The ability of the model to predict porewater pressures in pre-sheared, overconsolidated and unsaturated sands is confirmed. The model has considerable benefit in improving the understanding of the effects of different factors on liquefaction resistance.

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